Equations for Eclipsing Binaries

by Fraser Farrell

Last month I introduced you to some eclipsing binaries. Later this year, I will use our observations to deduce some facts about those star systems. But first, I'm going to outline the arithmetic involved…

For this exercise, let's imagine an "ideal" eclipsing binary system. Two spherical non-variable stars and a circular orbit exactly edge-on to our line of sight. One of the stars is a lot smaller and brighter than the other, and is traditionally labelled as the primary star. Less imaginative readers may refer to the diagrams of this ideal system, where I have shown the primary eclipse geometry as seen from Earth, and as seen from "above". For simplicity, I have shown the bigger secondary star as fixed, with the primary circling it—although in reality both stars would be orbiting the centre of gravity of the system.

The observed magnitude of the system remains at maximum until the primary star just begins to pass behind the secondary—position A (first contact). Then the magnitude falls, slowly at first, then more rapidly, then slowing again as the minimum is approached. The reason for this is because the primary star is seen as a (very, very tiny) disc of light which does not eclipse instantly. The primary has to travel "x" degrees along its orbit, which takes time. Minimum eclipse begins when the primary is just hidden (position B, or second contact) and ends when the primary begins its re-emergence (position C, or third contact). During this period the primary has travelled a further "y" degrees along its orbit. The rise to maximum mirrors the fall to minimum, and the eclipse is over when the primary reaches position D (fourth contact), after travelling another "z" degrees. It is evident from the above discussion that angles "x" & "z" are equal, and the entire eclipse involves the primary travelling (x+y+z) degrees around its orbit.

A secondary eclipse, when the primary star passes in front of the secondary, must occur and be of equal duration to the primary eclipse. However, if the secondary's surface is much dimmer (per square metre) than the primary's, then the secondary eclipse may be undetectable.

The lightcurve provides several times: "P" – the period of the orbit, "Px" – the time from maximum to minimum, "Py" – the time spent at minimum, and "Pz" – the time from minimum to maximum. "Px" is equal to "Pz" for this ideal system.

Now for the algebra! The ratios of the various times immediately provide the angles "x", "y" & "z" (in degrees):

$$x = 360 * Px/P,$$
$$y = 360 * Py/P,$$
$$z = 360 * Pz/P.$$

To obtain the ratios of the stars' diameters and their separation, assume "Dp" (the primary's diameter) as equal to 1 in the following equations.

If "P" is much larger than "Px", the primary star can be assumed to be "x" degrees wide as seen from the centre of the orbit. In this case, the approximate relationships:

$$R = 57.2958/x, \text{ and}$$
$$Ds = (y + 0.5 * (x+z))/x$$

will give "R" and "Ds" in terms of "Dp".

These approximations break down as "Px" becomes a significant fraction of "P"; i.e.: the stars are relatively close together. In this case, a more rigorous solution for "R" (at first contact) and "Ds" involves:

$$R = Dp / (2 * \cos(x + (y/2)) * \sin(x/2))$$
$$Ds = 1 + 2 * R * \sin(y/2)$$

If the orbital plane is not exactly edge-on to our line of sight, the result for "Ds" will be smaller than the true value. This can be confirmed if the orbital velocities are known, and the primary is calculated to move significantly more than its own diameter during periods "Px" and "Pz". The separation surface-surface is given by: R – ((Ds+Dp)/2).

Note that if this separation is relatively small, then the stars are probably tidally distorting each other. This can also be inferred from the lightcurve between eclipses (it will rise & fall slightly, rather than be flat as shown here). If this is the case, then the "diameter" of each component is approximate; and refers to the projected outline of each star upon the plane of the sky at the times of eclipse.

To put absolute dimensions in kilometres on our model, we need to know an orbital velocity, or be able to infer the sizes of each star by comparison with similar (known) stars.

The magnitude differences "Mp" & "Ms" provide clues to the relative luminosities of the stars. If the luminosities are "Lp" for the primary and "Ls" for the secondary then:

$$(Lp/Ls) = \text{antilog}(0.4 * Mp), \text{ or}$$
$$(Lp/Ls) = 1/(\text{antilog}(0.4 * Ms))$$

Complications to our simple model include:

(a) Either or both components being intrinsically variable. The eclipses are superimposed on the other variations at discrete intervals. Several dwarf novae including VW Hyi and OY Car exhibit this phenomenon.

(b) Annular eclipses. Although these also produce a flat-bottomed lightcurve, solving for the relative sizes of the stars as above soon reveals the true story! In this case, the calculated luminosities have to be corrected for the light from the uneclipsed fraction of the primary star.

(c) Partial eclipses—shown by a V-shaped lightcurve. In this case, "Py" equals zero, so the equations:

$$R = (x+z)/(720 * \cos(x)),$$
$$Ds \geq (2 * R * \cos(x))–1$$

should be used.

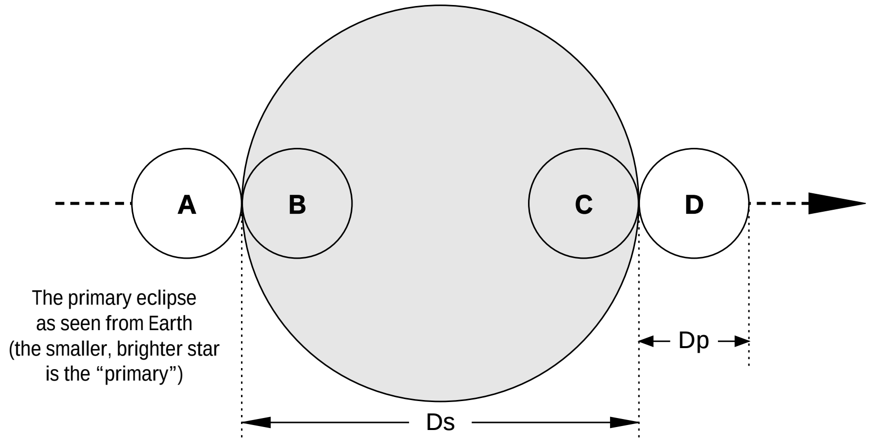
Note that "Ds" in this case is smaller than the true diameter. Also, the magnitude equations above can't be used. ▶

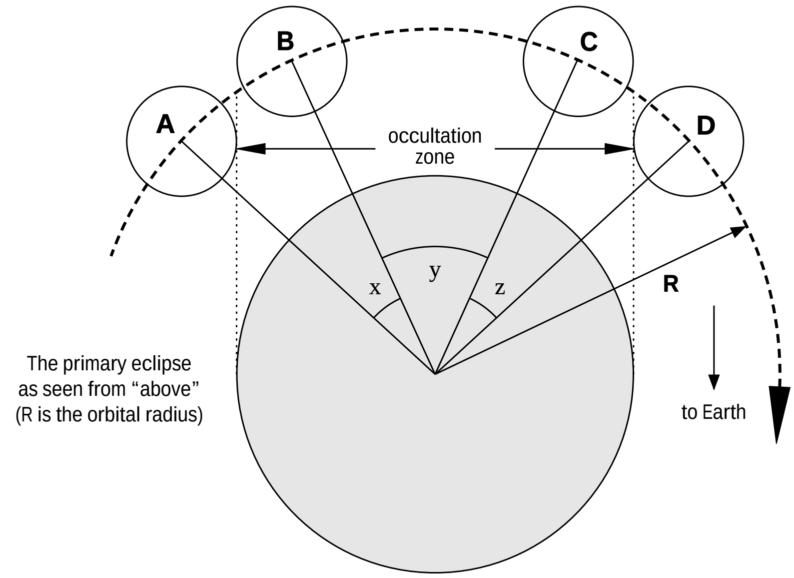
► (d) Elliptical orbits—revealed by non-identical “Px” and “Pz” periods. Solving rigorously for “R” on both sides of the eclipse (by using “z” in place of “x”) and using Kepler’s Second Law make it possible to estimate the size and eccentricity of the orbit. Long-term monitoring may reveal changes in the shape of the lightcurve due to precession of the orbit’s major axis.

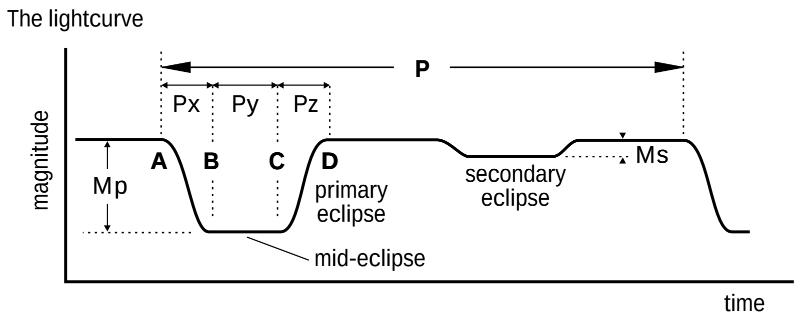
(e) Period changes. Causes include mass loss from either/both stars, tidal effects, or the influence of a third star/planet. Long-term monitoring is needed to detect period changes.

(f) Finally, limb darkening—the edges of the star discs being dimmer than the centres (due to light absorption in the stars’ atmospheres)—and starspots (like sunspots, but on a star). Detection requires photoelectric measurements at frequent intervals during the eclipse; followed by comparison of the results to theoretical models. Leave this job to professional astrophysicists…

Now all I need is lots of observations of eclipsing binaries to try out these equations on real lightcurves!







Variable Star Notices

• I can now be reached via the Internet. For general enquiries and observations my e-mail address is: **fraserf@dove.mtx.net.au**. If your message includes images, use PCX, GIF or BMP format please. Multiple files or very big files should be compressed in a ZIP archive before “attaching” it to your message. Don’t expect an instant answer, especially if the weather is clear and moonless!

◀ 5 Sayonara Hyakutake

agrees and we stand and admire this phenomena with pleasure.

Now, the big question, where is the comet? Trevor “discovers” it first with his 10×50s (no photography here!), hidden in the southern edge of the band of zodiacal light. The whole impact of this scene—the magnificent pre-dawn sky, the zodiacal light, the comet like a ghostly spectre within, the misty fields stretching away to Mt. Torrens silhouetted against the distant horizon, sheep and newborn lambs bleating nearby—will leave an everlasting impression. This is what Astronomy is about! (But don’t forget your thermal undies!)

One wonders who might have occupied this wonderful site thirty-two centuries ago, and having observed this celestial spectre, did they then go and create a dreamtime story of it to retell around the campfire for generations?

5:30 A.M. The TV crew arrives. Much discussion about telescopes, Astronomy, etc… We become TV stars and enjoy our two seconds of fame—the comet had months! The TV crew leaves.

The first flight from West Beach streaks to the southeast, beeline on a course above the glittering setting Canopus. Michael draws our attention to this man-made “comet”, its vapour trail brilliant against the background of stars. Southward from Mt. Torrens our nearest and most hospitable star impresses itself on the morning of May 17th 1996. Comet Hyakutake fades into oblivion. I wonder who will observe you from this site next time.

Sayonara Hyakutake, goodbye and thank you.